half-life of 4×10^7 yr for the virtual capture transition. The preliminary estimate[1] for X was 0.025 and was based on the assumption that log $(ft)=5.7$ for the 3/2− to 5/2− transition, and $\Gamma_\gamma=10^{-9}mc^2$ for the width of the $M1$ transition. The experimental results indicate that the product of those two probabilities is in fact smaller by a factor of ~10.

ACKNOWLEDGMENTS

The author is indebted to M. E. Rose and L. L. Foldy for proposing the experiment, to M. Kinsley for help with the chemical separations, to A. Schwarzschild for providing the least-square fitting program, and to J. Weneser, G. Scharff-Goldhaber, G. Emery, E. der Mateosian, and R. Marr for helpful discussions.

PHYSICAL REVIEW VOLUME 129, NUMBER 4 15 FEBRUARY 1963

Slightly Inelastic Proton-Deuteron Scattering*

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(Received 24 September 1962)

Measurements of the inelastic *p-d* cross section, in the region of very small momentum transfer, have been made at laboratory scattering angles of 5°, 10°, 15°, and 20°. The elastic *p-d* cross section has also been measured at these angles and compared with Postma's data. These measurements have been performed with a high-resolution magnetic spectrometer designed especially for this experiment. The *p-d* cross sections have been obtained by normalizing the *p-d* spectra to *p-p* spectra obtained by filling the same target with liquid hydrogen. The shape of the *p-p* spectrum, at a particular angle, was used to effect the separation of the inelastic *p-d* from the elastic *p-d* spectrum. The over-all energy resolution at small angles was about 0.75%.

A comparison of the elastic *p-d* data with the impulse-approximation calculation of Kerman, McManus, and Thaler yielded a value of Σ_t, the triplet amplitude sum, at the four angles measured. The singlet amplitude sum, Σ_s, was obtained by fitting Cromer's theory to the inelastic *p-d* cross section. The experimental values of the parameters Σ_s and Σ_t are compared with the predictions of the most recent phase-shift analyses. Σ_s appears to be particularly sensitive to the values of the $T=0$ amplitudes and thus experimental values of Σ_s may be useful in future phase-shift analyses.

INTRODUCTION

THE cross section for slightly inelastic proton deuteron scattering has been measured at four angles between 5° and 20° for an incident proton energy of 158 MeV. This report is concerned mainly with the experimental measurements whereas the theory and analysis of the data are dealt with in a companion article by Cromer.[1]

By the adjective slightly inelastic we refer to those collisions in which the incident proton transfers barely enough momentum to distintegrate the deuteron. Characteristically the momentum spectrum of the outgoing high-energy proton exhibits a rather small peak immediately below the elastic *p-d* peak. The shape of this slightly inelastic peak is determined mainly by the strong attractive potential operating in the final state (principally ^1S) of the recoiling two-nucleon system. The interest in studying collisions of this type derives from the fact that the cross section[1–3] can be shown to be rather sensitively dependent on the $T=0$ nucleon-nucleon scattering amplitudes.[4,5]

Earlier experimental work providing an important basis for the investigations reported here has been carried out at Harwell,[6,7] Uppsala,[8] and Harvard.[9,10]

GENERAL METHOD

The 158-MeV unpolarized proton beam was allowed to strike a small target of liquid deuterium. Protons scattered from the target were momentum-analyzed by a high-resolution magnetic spectrometer and detected by a simple array of scintillation counters. At each particular scattering angle the momentum spectrum of

* Supported by joint program of the Office of Naval Research and U. S. Atomic Energy Commission.
† Supported by the National Research Council of Canada.

[1] A. H. Cromer, following paper [Phys. Rev. **129**, 1680 (1963)].

[2] L. Castillejo and L. S. Singh, *Proceedings of the International Conference on Nuclear Forces and the Few-Nucleon Problem* (Pergamon Press Ltd., New York, 1960).

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[5] M. H. Hull, K. E. Lassila, H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **122**, 1606 (1961).

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[8] H. Tryen, G. Tibell, and Th. A. J. Maris, Nucl. Phys. **7**, 281 (1958).

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[10] A. Kuckes, R. Wilson, and P. F. Cooper, Jr., Ann. Phys. (N. Y.) **15**, 193 (1961).

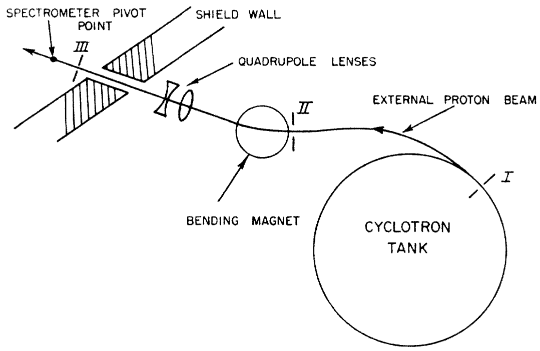

FIG. 1. Schematic view of the transport system used for the unpolarized protom bean.

protons scattered by deuterium nuclei was measured as a function of magnet current. The entire apparatus was calibrated by filling the same target with liquid hydrogen and making use of the accurately known *p-p* cross section.[11] In this manner the elastic and inelastic *p-d* cross sections have been measured at angles of 5°, 10°, 15°, and 20°.

The Beam

A schematic drawing of the unpolarized proton beam is shown in Fig. 1. The bending and quadrupole magnets served to focus the proton beam to a $\frac{3}{8}$-in. diameter spot at the experimental target. The energy of the beam was defined to about 1 MeV by means of defining slit I, immediately prior to the magnetic channel of the cyclotron.[12] Slit II, placed about 18 inches ahead of the target chamber, removed the halo from the beam. The intensity of the beam under these conditions was ordinarily about 10^9 protons per sec.

Target and Scattering Chamber

The scattering chamber is illustrated in Fig. 2. The lengths of vacuum pipe together with the antiscattering baffles reduced the nontarget-associated background to less than 0.1% of the real counting rate.

The target cups were constructed of 2-in. high cylinders of 1-mil Mylar foil. For measurements at 10°, 15°, and 20° the diameter of the cup was chosen to be $\frac{1}{2}$ in. The diameter had to be fairly small owing to the absolute necessity of obtaining high resolution with the target spectrometer system. At 5° the diameter was increased to $\frac{3}{4}$ in. in order to keep the background due to the Mylar and aluminum radiation shield less than 25% of the real rate.

Beam Monitors

A nonsaturating ionization chamber (76 cm of helium plus 4 cm of nitrogen) monitored the beam intensity. The multiplication factor was 250, as determined by a Faraday Cup.

The lateral wandering of the beam with respect to the narrow vertical target cylinder was monitored continuously by a split ionization chamber.[13] The sensitivity of this device was adjusted so that lateral displacements of the beam exceeding ±0.05 in. were very easily detected. By requiring that the beam be stable to ±0.05 in. in the horizontal plane, the random error of this type was held to about ±2%.

The Magnetic Spectrometer

The gross features of the spectrometer are illustrated in Fig. 3 and the important mechanical and physical parameters are listed in Table I. The spectrometer was ordinarily used with a $\frac{1}{2}$-in. wide target carefully positioned above the pivot point. The focal plane of this spectrometer was occupied by an array of 5 juxtaposed scintillators, each having a width of 0.078 in. and separated from its neighbor by only a few mils of aluminum foil. Two $\frac{1}{2}$-in. wide scintillators, mounted 9 and 12 in., respectively, behind the focal-plane array were connected in fast coincidence with each of the 5 momentum-defining counters. The electronic circuits

TABLE I. Parameters of the spectrometer magnet system.

Radius of curvature	56 in.
Angle of bend	80°
Gap	2 in. (vertical)×4 in. (radial)
Weight	12 tons
Object distance	100 in.
Magnet spacing	5 in.
Focal length	38.5 in. (calculated), 38.0 in. (measured)
Solid angle	Approximately 10^{-4} sr
Resolving power with $\frac{1}{2}$-in. target	560 (calculated) 530 (measured)

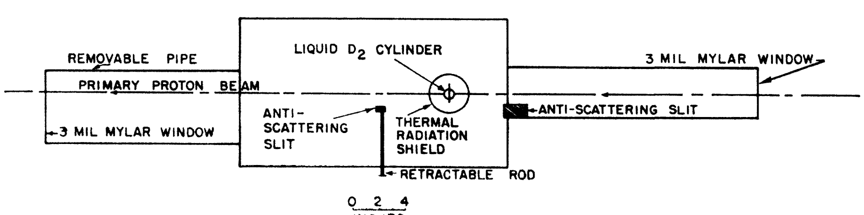

FIG. 2. The target chamber.

[11] C. Caverzasio, K. Kuroda, and A. Michalowicz, J. Phys. Radium 21, 319 (1960).
[12] G. Calame *et al.*, Nucl. Instr. 1, 169 (1957).
[13] B. Gottschalk, A. M. Koehler, and D. J. Steinberg, Rev. Sci. Instr. 32, 744 (1961).

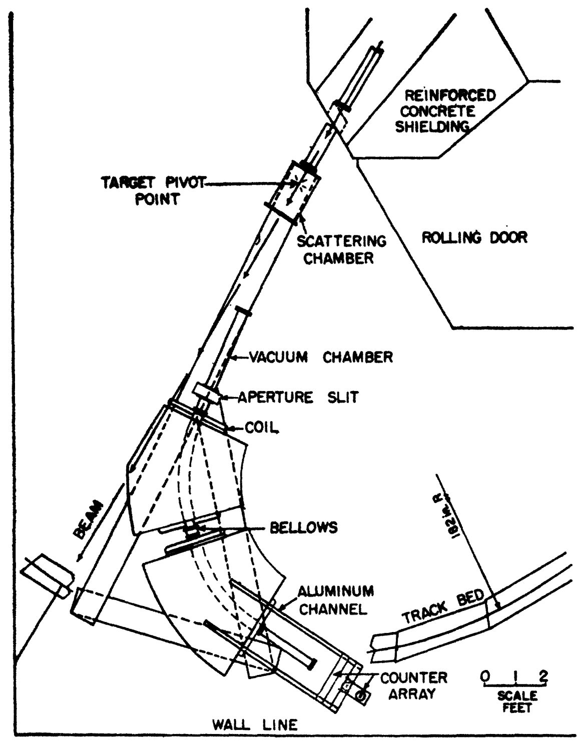

FIG. 3. The magnetic spectrometer.

used were of the type designed by Gabriel, Garwin, and York.[14]

The magnetic field of the spectrometer was measured with a rotating coil device. To within the accuarcy of the field measurements (about 1%) no evidence for saturation effects could be discerned in the vicinity of 13 000 G, the nominal field value required for the experiment. The current was regulated to about $\pm 0.01\%$, a stability sufficient to make feasible a high-resolution experiment such as slightly inelastic proton scattering.

The intrinsic energy resolution of the spectrometer was about 0.3% as determined from the separation of an elastic peak in adjacent momentum-defining counters. This resolution was small enough that the over-all experimental resolution could be attributed mainly to the spread in energy of the incident beam. With increasing scattering angle, a slight kinematic broadening of the elastic peak was observed in the case of p-p scattering.

EXPERIMENTAL PROCEDURE

The beam was focused to a $\frac{3}{8}$-in. diameter spot at a point directly above the stationary pivot of the spectrometer. Having secured the desired spatial path for the beam, the split ionization chamber was adjusted to monitor excursions about this position by amounts equal to or greater than ± 0.05 in. in the horizontal plane.

In order to facilitate certain instrumental checks, a thin carbon target was more convenient than the liquid target cryostat. Thus, protons scattered from the carbon target were allowed to traverse the spectrometer and pass through the array of counters. Five triple timing curves were performed, generally with a resolving time set to approximately 30 nsec. Having inserted the requisite delays, the discriminator plateau curve was measured for each channel and each discriminator was adjusted to operate at the center of its plateau. Then each photomultiplier was set to the center of its voltage plateau, the width of which was generally about 100 volts for all counters.

A most important preliminary measurement was the determination of the over-all experimental resolution. This test was generally carried out with a carbon target $\frac{1}{16}$ in. thick and $\frac{1}{2}$ in. wide. If Δp be the full width of the elastic peak at half maximum, the momentum resolution is (nonrelativistically),

$$\Delta p/p = \Delta i/i = \tfrac{1}{2}\Delta E/E,$$

where i is the spectrometer current and E is the proton kinetic energy. As previously remarked, the resolution was limited mainly by the energy spread of the incident beam. To reduce this spread in energy a set of slits within the cyclotron tank was adjusted to accept an energy bite, ΔE, of about 1 MeV.[12] With the beam conditions as described and an aperture slit in front of the spectrometer, the elastic peaks obtained were symmetric and substantially free of large low-energy tails. The energy resolution obtained in the actual experiment with p-d elastic peaks was about $\frac{3}{4}\%$.

Hydrogen and Deuterium Data

For the actual data runs a conventional liquid hydrogen/deuterium cryostat was carefully mounted and aligned so that its center was directly above the pivot point of the spectrometer.

In this part of the experiment the fluctuations in beam position as well as the stability of the spectrometer current were continuously monitored. As a useful check during these runs, the number of protons contained in an elastic peak was used as the only meaningful measure of reproducibility. From run to run for either hydrogen or deuterium at a given scattering angle, the number of protons in an elastic peak was constant to within $\pm 3\%$, this limit being imposed by fluctuations in the spectrometer current.

These effects would have been less worrisome had the counter array been large enough to scan an entire elastic peak in a single counting period. We note, however, that the process of summing the counts from the various momentum channels reduced the uncertainty quoted above to about $\pm 1.5\%$.

[14] R. Gabriel, E. L. Garwin, and C. M. York, Nucl. Instr. 5, 247 (1959).

Accidental Coincidences

Because of the comparatively poor duty cycle of the cyclotron, with an associated high instantaneous counting rate, a careful measurement of the number of accidental coincidences was made at each scattering angle.

To make this measurement, each small counter was delayed by an additional 22 m of RG62U coaxial cable. This length introduced a delay equal to the temporal separation of two fine structure bursts of intensity from the cyclotron. This time delay was more than twice the resolving time of the coincidence circuit; consequently a triple coincidence circuit output with this arrangement was necessarily random.

With the target full of liquid deuterium, the spectrometer current was set at a value known to correspond to the maximum of the elastic peak at the particular scattering angle under study. The number of random coincidences was measured by switching on the scalers for 75 ionization chamber counts, a counting period which would ordinarily yield about 4000 proton counts at the maximum of the elastic *p-d* peak. The number of random coincidences never exceeded one in any channel. To be certain of these measurements, the delay cables were checked for broken connections and for anomolously large attenuation of the pulses from the small counters. No such undesirable effects were discovered.

Background Measurements

Background contributions in this experiment resulted from the 1-mil Mylar target cup and its concentric $\frac{1}{4}$-mil aluminum radiation shield. Fortunately, at 10°, 15°, and 20°, the spectrometer was able to resolve the background peak from the elastic *p-d* and *p-p* peaks. As a result, the background contamination in these peaks was about 1%. At each angle, the measurements were extended over the entire range of the inelastic *p-d* spectrum. At 5°, the magnitude of the background was larger than the background at wider angles. Moreover the spectrometer was quite unable to resolve the background peak from the *p-d* or *p-p* peaks. Thus the background contamination at 5° was about 25% of the *p-p* peak (area) and about 12% of the *p-d* peak (area). These numbers agreed well with the results of approximate calculations of the ratio of true to background counts, using the known chemical compositions of the radiation shield and the Mylar.

CORRECTIONS TO THE DATA

Dead-Time Corrections

The combination of slow scalers and the relatively poor duty cycle of the cyclotron produced significant counting losses in some of the data. The correction was obtained from Cormack's formula for the case of a pulsed beam[15]:

$$n_{\text{true}} = n_0/(1 - n_0\tau/2tD),$$

where n_0 is the number of observed counts in t seconds, τ is the dead time of the scaler, and D is the duty cycle of the cyclotron.

The manufacturer's quoted dead time of 10^{-5} sec is probably not very accurate. However, because of the smallness of the dead-time correction, a substantial uncertainty in τ is tolerable. With a duty cycle of 1/250, the corrections were typically about +3% at the maximum of the elastic *p-d* peak and about +0.5% on the inelastic spectrum. The dead-time correction was also applied to the background spectrum at 5°; at other angles, where the background was small, the dead-time correction was unnecessary.

Dispersion Effect

All the spectra were corrected for the so-called dispersion effect[16] wherein the momentum acceptance Δp of a magnetic spectrometer varies linearly with the momentum of the particle. These corrections were typically less than a few percent.

Background Subtraction

The background spectrum could not be immediately subtracted from the target-full spectrum. Before subtraction, the background spectrum had to be displaced towards lower energy by an amount equal to the energy loss in traversing the target liquid. Owing to the intensity distribution across the circular face of the target cylinder, the effective thickness, and hence the energy loss, could not be calculated to an accuracy better than ±10%.

This difficulty was avoided by appealing to the kinematics of elastic scattering. For example at 5°, the recoil energy in *p-p* scattering is 1.283 MeV whereas in *p*-carbon scattering (Mylar cylinder) the recoil energy is 0.008 MeV. Thus the background peak was located 1.275 MeV above the *p-p* peak.

METHOD OF SEPARATING THE INELASTIC SPECTRUM

The resolution of the apparatus was insufficient to isolate completely the inelastic spectrum from the elastic *p-d* peak. An indirect method was used to effect the separation; the explanation of the method will be much clearer if reference is made to Fig. 4, where the separation is illustrated for data at 10°.

The corrected data for the various channels were combined to produce composite *p-p* and *p-d* spectra which were plotted against spectrometer current. The basic idea was to extrapolate the tail of the elastic *p-d*

[15] A. M. Cormack, Nucl. Instr. **15**, 268 (1962).

[16] *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), p. 54.

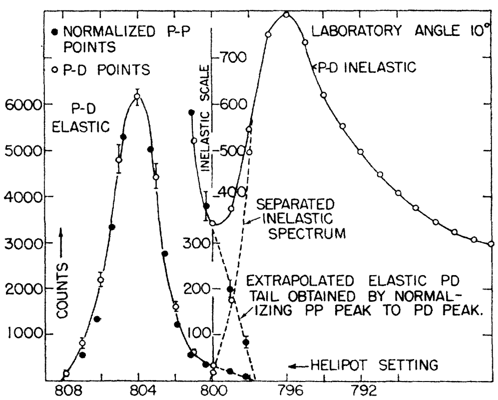

FIG. 4. Illustrating the method of separating the inelastic *p-d* spectrum from the elastic spectrum.

spectrum, using the elastic *p-p* spectrum as a model. Because of the difference in kinematics involved in *p-p* and *p-d* collisions, the hydrogen peaks, at a given angle, were broader than the deuterium peaks. Therefore, a simple amplitude normalization between the two peaks was not sufficient.

Denote the peak amplitude and width at half maximum by A and w, respectively. Then the desired normalization was accomplished by multiplying all *p-p* ordinates by A_D/A_H and all *p-p* abscissas by w_D/w_H. The extrapolated tail thereby obtained was also plotted on an expanded scale (see Fig. 4). The separation was completed by subtracting this tail from the curve representing the combined effects of elastic and inelastic events.

In addition to the *p-p* normalization method, two other tails were appended to the *p-d* peak by simple linear extrapolation. These two extrapolations were chosen to approximate reasonable maximum and minimum low-energy tails. The areas of these tails were about 25% larger and smaller, respectively, than the area of the tail obtained by *p-p* normalization. Hence the uncertainty on the *p-p* method is perhaps ±25%; since the area of a tail is only about 2% of the total area, the area of the elastic peak is uncertain by about ±0.5% as result of the separation procedure.

TREATMENT OF RANDOM ERRORS

The statistical error for each point on a spectrum was compounded of four separate contributions:

1. Spectrometer current fluctuations were allowed for by computing for each point $f(i)$ the quantity $\Delta f = f'(i)\Delta i$, where $f'(i)$ was the slope at current i.

2. Counting statistics gave a contribution of $\pm f^{1/2}$.

3. Target thickness fluctuations due to lateral wandering of the beam were limited to about $\pm 0.02f$ by use of the split ionization chamber.

4. An additional random error of $\pm 0.02f$ represented fluctuations in the ionization chamber used to monitor the beam intensity. The standard deviation on an individual point was computed according to the formula,

$$[f+(\Delta f)^2+2(\beta f)^2]^{1/2},$$

where $\beta = 0.02$.

When applied to a typical *p-p* spectrum at a scattering angle of 10°, the uncertainty in the area under the peak amounted to ±2.1%. A composite spectrum, obtained by compounding four such spectra, has an uncertainty in the area of about ±1%. Similar statements apply to the *p-d* data. Consequently, the final error in the derived elastic and inelastic *p-d* cross sections is dominated by the error in the *p-p* cross sections of Caverzasio *et al.*,[11] about ±5% at small angles.

To illustrate this latter point we present in Table II a calculation of the elastic *p-d* cross section.

THE ELASTIC *p-d* CROSS SECTION

The elastic *p-d* cross section has been obtained by normalizing the elastic deuterium peak to the hydrogen peak. Thus if A_D and A_H represent the areas under the elastic *p-d* and *p-p* peaks, respectively, it follows that,

$$\left(\frac{d\sigma}{d\Omega}\right)_{pd} = \frac{A_D}{A_H}\frac{(\rho t)_H}{(\rho t)_D}\frac{M_D}{M_H}\left(\frac{d\sigma}{d\Omega}\right)_{pp}, \tag{1}$$

where ρ is the density, M is the atomic mass number, and t is the effective target thickness. The quantity t can be calculated from the geometry of the target provided the spatial distribution of the incident beam intensity is known. Since the same target was used for both hydrogen and deuterium data, the effective target thickness cancels out in Eq. (1). The appropriate densities were taken from the compilation of Chelton and Mann.[17] The values of the *p-p* cross section were those recently measured by Caverzasio, Kuroda and Michalowicz.[11]

THE INELASTIC *p-d* CROSS SECTION

The inelastic *p-d* cross section is given by $d^2\sigma/d\Omega dp = \alpha N(\theta,p)$, where $N(\theta,p)$ is the corrected inelastic spectrum and α is a constant given by the ratio $(d\sigma/d\Omega)_{pd}/$

TABLE II. The evaluation of the elastic *p-d* cross section is illustrated for a laboratory scattering angle of 5°. Note that the dominant error is that of the *p-p* cross section.

Quantity	Value	Error
M_D/M_H	1.998	0
t_H/t_D	1.005	+0.1%
ρ_H	70.8 g/liter	±0.5%
ρ_D	169 g/liter	±0.5%
A_D	5670	±1.1%
A_H	2604	±1.5%
$(d\sigma/d\Omega)_{pp}$	16.96 mb/sr	±4.0%
$(d\sigma/d\Omega)_{pd}$	31.05 mb/sr	±4.5%

[17] D. B. Chelton and D. M. Mann, University of California Radiation Laboratory Report UCRL-3421, 1956 (unpublished).

A_D, where these symbols have their previous definitions. The differential cross section per MeV is given by $d^2\sigma/d\Omega dE=(\alpha\beta/c)N(\theta,p)$, where β is the ratio of the speed of the scattered proton to the speed of light, c.

RESULTS

The Elastic *p-d* Cross Section

In this section the results of the elastic *p-d* cross section measurements are presented in Table III for two experimental runs, denoted by I and II. Runs I and II have been normalized to the *p-p* cross section measurements of Caverzasio *et al.*[11] The errors quoted by Caverzasio *et al.*[11] are the principal uncertainty in the elastic *p-d* cross sections listed in column IIb. To illustrate this point column II is divided into two parts: IIa includes the error of the present experiment only, the *p-p* cross sections being regarded as exact numbers; IIb includes the error of the present experiment combined with the *p-p* errors quoted by Caverzasio *et al.*[11]

Discussion of the Results for the Elastic Cross Section

The procedures and apparatus described in earlier sections were those of run II. Run I differed from II in that no monitoring devices were used to determine the fluctuations in spectrometer current and in the position of the incident beam. Without this information it is difficult to estimate the error in the cross sections of run I. Furthermore, the *p-p* and *p-d* spectra were measured only once at a given angle, most of the available time having been spent on reducing the energy spread of the external proton beam. Accordingly, the results of run I are assumed to be uncertain by ±10%, an over-all error about twice that of run II.

Since the same *p-p* cross sections were used to normalize I and II and because the *p-p* error is the dominant error in II, the difference between these two sets of measurements is due to a systematic error of about 10% in I.

In view of the substantial errors attached to the results of run I and Postma's results,[9] the relatively good agreement between these independent measurements is probably accidental. It is evident that run II is consistently low with respect to Postma's data. However, at the three angles considered, the ratio and the mean ratio are within one standard deviation of unity.

The discrepancy between run II and Postma's data would be considerably reduced if Palmieri's *p-p* cross sections had been used for normalization.[18] However, the *p-p* cross sections of Caverzasio *et al.*[11] have been used for normalization because it is felt that these results are superior to Palmieri's. It is probable that if

[18] J. N. Palmieri, A. M. Cormack, N. F. Ramsey, and Richard Wilson, Ann. Phys. (N. Y.) **5**, 299 (1958).

TABLE III. Values for the elastic *p-d* cross section in mb/sr at 158 MeV (I, IIa, IIb) and at 147 MeV (III, Postma). The errors shown in Column IIa are those for this experiment only while IIb includes the normalization error associated with the *p-p* cross section.

Laboratory angle	I	IIa	IIb	III
5°	not measured	31.05±0.65	31.05±1.40	34.15±3.09
10°	40.66±4.10	36.44±0.80	36.44±2.22	39.00±3.53
15°	24.49±2.45	22.29±0.46	22.29±0.82	24.35±3.77
20°	13.36±1.24	not measured	not measured	12.30±1.05

Palmieri's *p-p* cross sections are high by 8%, as they are with respect to those of Caverzasio *et al.*,[11] then Postma's results are high by the same factor. Palmieri has recently proved experimentally that the discrepancies are the result of a 10% calibration error in the Faraday cup used in his work and Postma's.

The value of $\Sigma_t(q)$ can be obtained from the calculation of Kerman, McManus, and Thaler.[3] They have shown that, in impulse approximation, the elastic *p-d* cross section in the laboratory system is

$$(d\sigma/d\Omega)_{\text{elastic}}=4|F_D(q)|^2\Sigma_t, \qquad (2)$$

where $F_D(q)$ is the form factor for the deuteron and where

$$\Sigma_t=|A|^2+(5/3)|C|^2+(2/3)(|B|^2+|E|^2+|F|^2), \qquad (3)$$

with $A=3A_1+A_0$, etc. The subscripts refer to the isotopic spin one and zero states, respectively. The quantities A, B, C, etc. are the coefficients of the nucleon-nucleon scattering matrix.[1,3,19]

Using a form factor derived from the Hulthén wave function,[20] the value of $\Sigma_t(q)$ was obtained from the measured *p-d* elastic cross sections of run II at 5°, 10°, and 15° and run I at 20°. These measured valued of Σ_t are compared with the phase-shift predictions of three groups of authors:

1. Gammel and Thaler.[21] These original *p-p* and *n-p* phase shifts at 155 MeV (from which the amplitudes A,

TABLE IV. Comparison of the values of Σ_t in mb obtained from the analysis of elastic proton-deuteron scattering with the phase-shift predictions of Breit *et al.*,[a] Prenowitz[b] and Kerman, McManus and Thaler.[c]

Laboratory angle	Elastic *p-d*	YLAM YLAN3M	Prenowitz YLAN3M	KMT
5°	7.82±0.35	6.29	7.18	6.82
10°	11.52±0.70	10.89	11.49	11.54
15°	9.79±0.37	11.32	11.94	11.59
20°	8.59±0.80	10.11	11.02	9.76

[a] See reference 5.
[b] See reference 21.
[c] See reference 3.

[19] L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).
[20] M. J. Moravcsik, Nucl. Phys. **7**, 113 (1958).
[21] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957).

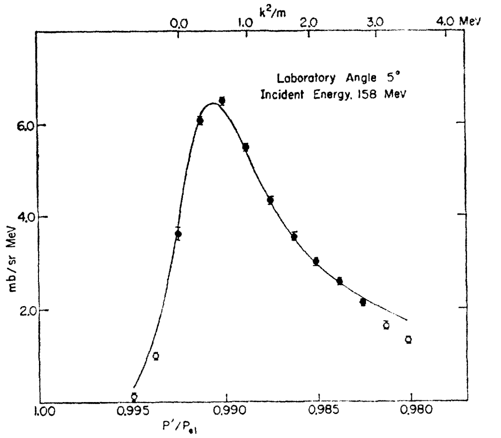

FIG. 5. The inelastic p-d cross section is shown for a laboratory scattering angle of 5°. The solid curve is Cromer's fit[1] to the experimental data. The cross sections are in units of mb/sr MeV and are plotted against the ratio of the momentum of an inelastic proton to the momentum of an elastic proton at the various angles. The upper abscissa is in units of MeV and represents the corresponding energy in the center of mass of the recoiling two-nucleon system.

B, etc. are derived) are now presumably outdated, but are included here for the sake of completeness.

2. Breit and collaborators.[4,5] These authors have recently completed an energy-dependent phase-shift search, the results of which are superior to those of (1) in the prediction of experimental results. The prediction of Σ_t by these authors, based on their best p-p solution (denoted by YLAM) and n-p solution (YLAN3M) at 140 MeV, are listed in column II;

3. The n-p amplitudes of Breit *et al.* (YLAN3M) were combined with the p-p amplitudes at 147 MeV of

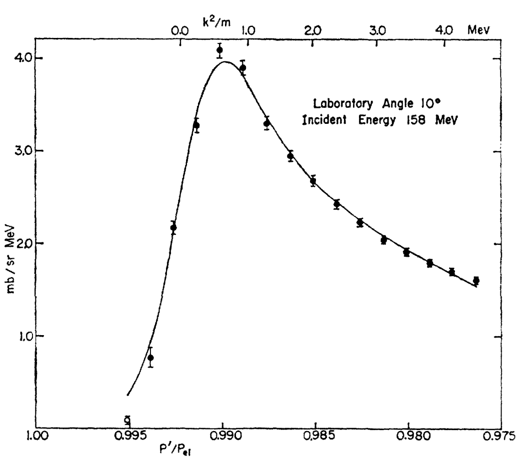

FIG. 6. The inelastic p-d cross section is shown for a laboratory scattering angle of 10°. The solid curve is Cromer's fit[1] to the experimental data. The cross sections are in units of mb/sr MeV and are plotted against the ratio of the momentum of an inelastic proton to the momentum of an elastic proton at the various angles. The upper abscissa is in units of MeV and represents the corresponding energy in the center of mass of the recoiling two-nucleon system.

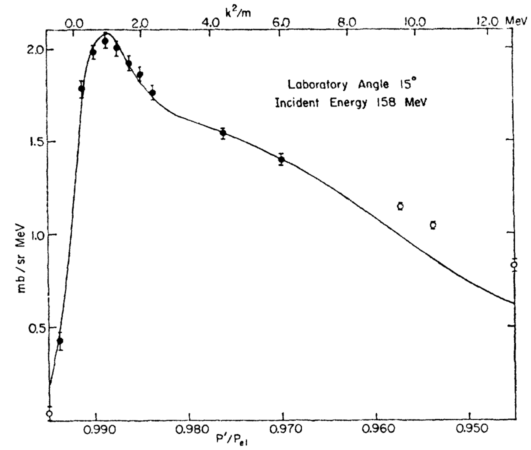

FIG. 7. The inelastic p-d cross section is shown for a laboratory scattering angle of 15°. The solid curve is Cromer's fit[1] to the experimental data. The cross sections are in units of mb/sr MeV and are plotted against the ratio of the momentum of an inelastic proton to the momentum of an elastic proton at the various angles. The upper abscissa is in units of MeV and represents the corresponding energy in the center of mass of the recoiling two-nucleon system.

Palmieri and Prenowitz,[22] of the Harvard Cyclotron Laboratory.

For the comparison of measured and predicted values of Σ_t one should, in principle, use the predicted, rather than the measured, p-p cross section to normalize the p-d data. In this work the p-p cross sections of Caverzasio *et al.*[11] have been used for normalization

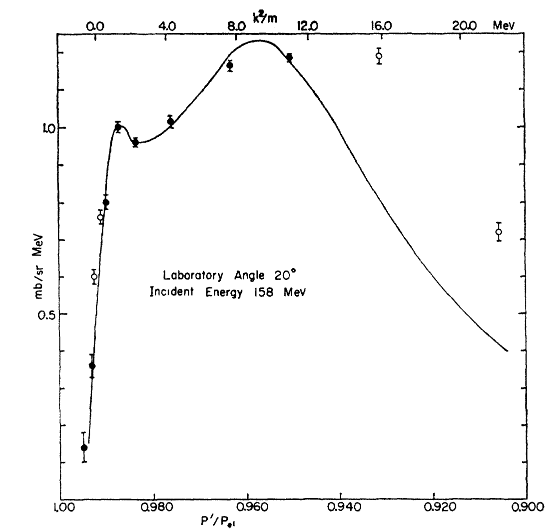

FIG. 8. The inelastic p-d cross section is shown for a laboratory scattering angle of 20°. The solid curve is Cromer's fit[1] to the experimental data. The cross sections are in units of mb/sr MeV and are plotted against the ratio of the momentum of an inelastic proton to the momentum of an elastic proton at the various angles. The upper abscissa is in units of MeV and represents the corresponding energy in the center of mass of the recoiling two-nucleon system.

[22] J. N. Palmieri and E. E. Prenowitz (to be published).

but the error attached to the Σ_t values of Column IV, Table IV, are those of this experiment only. In other words, the *p-p* cross section has again been treated as an exact number because, strictly speaking, the error attached to *p-p* data should not enter into the comparison of experimental and predicted values of Σ_t.

THE INELASTIC CROSS SECTION

The elastic *p-d* cross sections of run II were used to normalize the inelastic spectra at 5°, 10°, and 15°; at 20° the cross section of run I had to be used since run II did not include measurements at this angle.

Figures 5, 6, 7, and 8 display the inelastic cross sections at the four angles. The inelastic cross sections in units of mb/sr MeV are plotted as functions of the ratio P'/P_e, where P_e is the momentum of an elastically scattered proton and P' is the momentum of an inelastically-scattered proton. The error bars attached to each point of these figures represent the total error, that is the error of this experiment combined in quadrature with the error of the *p-p* cross section used in normalization.

Cromer[1] has obtained from the data the value of $\Sigma_s(q)$, the singlet amplitude sum, defined by

$$\Sigma_s=\tfrac{1}{4}[|B|^2+|C|^2+|E|^2+|F|^2], \qquad (4)$$

where $B=B_1-B_0$, etc. The subscripts one and zero refer to isotopic spin one and zero, respectively. The values of Σ_s are displayed in Table V and are compared with the phase-shift predictions.

TABLE V. Comparison of the values of Σ_s in mb found from the analysis of inelastic proton-deuteron scattering with the predictions of the phase-shift solutions of Breit *et al.*,[a] Prenowitz,[b] and Kerman, McManus and Thaler.[c]

Laboratory angle	Inelastic *p-d*	YLAM YLAN3M	YLAM Prenowitz	KMT
5°	12.17±0.25	10.11	10.36	10.5
10°	11.50±0.70	8.02	7.78	7.79
15°	10.32±0.54	6.16	6.26	5.70
20°	6.70±0.45	4.96	5.43	4.91

[a] See reference 5.
[b] See reference 21.
[c] See reference 3.

CONCLUSION

The substantial discrepancies between the phase-shift predictions for $\Sigma_s(q)$ and those obtained from the data reported here are well beyond those expected on the basis of experimental errors only. The various approximations involved in the analysis are probably responsible for part of the discrepancy. It is important to note that the phase-shift solutions YLAM and YLAN3M have statistical uncertainties[4,5] which affect the accuracy of the predicted values of Σ_s and Σ_t.

Since $\Sigma_s(q)$ is particularly sensitive to the $T=0$ amplitudes, which amplitudes are not yet well determined, at least some of the discrepancy between the predicted and measured values of this quantity may be real. It is our hope that the values of $\Sigma_s(q)$ may be useful in future phase-shift analyses.

ACKNOWLEDGMENTS

We wish to thank all members of the Cyclotron Laboratory for their assistance at various times. We are particularly indebted to Andreas Koehler who suggested the double magnet system and designed the current stabilizer for it. The analysis of the data has been carried out with the invaluable help and advice of Dr. A. H. Cromer.